CHANGE-OVER DESIGNS

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1. Introduction

The experiments in which each experimental unit receives some or all of the treatments, one at a time, over a certain period of time are known as change-over designs. These designs are also called cross-over designs, switch-over trials, time-series designs, before-after designs, repeated measurements designs, designs involving sequences of treatments *etc*. These designs have been used advantageously in several fields of research, notably in nutrition experiments with dairy cattle, clinical trials in medical research, psychological experiments, long--term agricultural field experiments and bio-assays.

A change-over design with four treatments A, B,C and D, in four periods and four experimental units is as given below:

	Experimental Units			
Periods	1	2	3	4
1	A	B	C	D
2	D	A	B	C
3	B	C	D	A
4	C	D	A	B

A large volume of literature in Design of Experiments is devoted to the study of change-over designs. These designs have been studied by several research workers; Williams (1949), Patterson (1952), Patterson and Lucas (1962), Berenblut (1964), Saha (1970), Sharma (1974) *etc* are a few among them.

2. Need for Change-Over Designs

There are several situations, where it is essential to go for the change-over designs. Some of these are:

(i) Due to budget constraint, the experimenter has to use each experimental unit for several tests.

(ii) In some experiments, the treatment effects do not have a serious damaging effect on the experimental units and hence these units can be used for successive occasions.

(iii) In some experiments, the experimental units are human beings or animals and often the nature of the experiments is such that it calls for special training over a long period of time. Therefore, due to time limitations one is forced to use these experimental units for several tests.

(iv) When one of the objectives of the experiments is to find out the effect of different sequences as in drug, nutrition or learning experiments.

(v) Sometimes the experimental units are scarce, therefore, experimental units have to be used repeatedly.

3. Residual Effects

The peculiarity of a change-over design is that any treatment applied to a unit in a certain period influences the responses of the unit not only in the period of its application but also leaves residual effects in the following periods. These residual effects or carry-over effects may be of different magnitudes. Residuals, which persist only for one succeeding period, are called first order residual effects or simply, first residual effects. In general, the k^{th} order residual effect is one, which persists up to k successive periods. This feature of residuals is the distinguishing feature of change-over designs.

Change-over designs are generally capable of providing treatments comparisons of high precision because they eliminate the difference among experimental units from the error variation. This advantage, of course, is offset by the possible complications that arise in view of the presence of residual effects. One of the ways of getting rid of the complications (in analysis) due to the presence of residual effects is to insert a rest period between successive experimental periods, such that residual effects, if any, may wean out during the rest period. However, in some cases it is not possible to have the rest periods. Alternatively, provision for the separation of direct and residual effects can be made in the experimental design by a suitable choice of treatment sequences.

The simplest type of change-over design is a Latin square design or a set of squares with rows representing periods of time and columns the experimental units. Provided that there are no carry-over or residual effects and experimental conditions are not such that treatment effects vary to any marked extent from one period to another, the ordinary analysis of Latin square designs can be used. But it is always desirable to allow for the possibility that residual effects exist. An allowance for residual effects can be made in the design and by introducing additional constants for residual effects in the model, the analysis of the data can be carried out.

4. Balanced Change-Over Designs

The analysis is made easier if each treatment is preceded equally often by every other treatment. Designs having this property have been called *balanced* with respect to first residual effects. Or in other words, a change-over design permitting the estimation of first order residual effects, is called balanced if the variance of any elementary contrast among the direct effects is constant, say α, and the variance of any estimated elementary contrast among the residual effects is also constant, say β. Here, α and β may not be equal. If $\alpha = \beta$, then these designs are known as *totally balanced change-over designs*. Designs in which each treatment is preceded by every other treatment as well as by itself equally often are called *strongly balanced change-over designs*.

5. Uniform Change-Over Design

A change-over design is called uniform on periods if each treatment occurs in each period the same number of times, say λ_1. A necessary condition for this to hold is that the number of units, $n = \lambda_1 v$, v being the number of treatments.

A design is called uniform on units if each treatment is applied to each experimental unit the same number of times, say λ_2. This can occur only if the number of periods, $p = \lambda_2 v$.

A design is called uniform if it is uniform on both periods and units.

A Williams square COD can be considered as a uniform COD. For v = 5 treatments a Williams square COD is as follows:

	First Latin square					Second Latin square				
	Experimental units					Experimental units				
Periods	I	II	III	IV	V	I	II	III	IV	V
1	1	2	3	4	5	5	1	2	3	4
2	5	1	2	3	4	1	2	3	4	5
3	2	3	4	5	1	4	5	1	2	3
4	4	5	1	2	3	2	3	4	5	1
5	3	4	5	1	2	3	4	5	1	2

6. Model

Let a COD be represented as COD(v, p, n) where v treatments are arranged in p periods and n experimental units. Considering the first residual effects of treatments, the additive fixed effects model can be written as

$$Y_{hijk} = \mu + \pi_h + \tau_i + \rho_j + \psi_k + \varepsilon_{hijk}\,; \qquad \ldots(1)$$

$$h = 1, \ldots p;\ i, j = 1, \ldots, v;\ k = 1, \ldots n;$$

where Y_{hijk} = observation from the k[th] experimental unit in the h[th] period when treatment i is applied to it and is preceded by the j[th] treatment in (h-1)[th] period (h>1) and μ , π_h , ψ_k , τ_i , and ρ_j represent the general mean, effect of the h[th] period, effect of the k[th] unit, direct effect of treatment i and residual effect of the treatment j, respectively, ε_{hijk} are random errors assumed to be identically and independently distributed with N(0, σ^2). $\rho_j = 0$ for all values of j for the observations in the first period. The same model may be expressed in matrix notation as:

$$\mathbf{y} = \mu\,\mathbf{1} + \mathbf{P}\,\pi + \mathbf{T}\tau + \mathbf{R}\,\rho + \mathbf{S}\,\psi + \varepsilon, \qquad \ldots(2)$$

with E (ε) = 0 and E ($\varepsilon\,\varepsilon'$) = σ^2 **I,** where

Y = np × 1 vector of observed responses,
π = p × 1 vector of period effects,
τ = v × 1 vector of direct effects of treatments,
ρ = v ×1 vector of residual effects,
ψ = n × 1 vector of sequences/unit effects,
1 = np × 1 vector of unities,
P = np × p design matrix of observations vs. periods,
T = np × v design matrix of observations vs. direct effects of treatments,
R = np × v design matrix of observations vs. residual effects of treatments,
S = np × n design matrix of observations vs. experimental units,
$\varepsilon = (\varepsilon_1, \ldots, \varepsilon_{np})$ is np × 1 vector of random errors.

7. Extra-Period Change-Over Designs

The balanced change-over designs may not be desirable always because:
(i) they do not estimate direct and residual effects of treatments independently, for the estimates are correlated,
(ii) they give less precise estimates of residual effects than of direct effects.

Designs that give independent estimates of direct and residual effects, of approximately equal precision, are obtained by adding an extra period to the original design. In the new final period, the treatments that were applied in the previous final period are repeated, and these designs are called *extra-period balanced change over designs*. The following is an extra-period balanced change-over design for 3 treatments A, B and C:

	Experimental Units					
Periods	1	2	3	4	5	6
1	A	B	C	A	B	C
2	B	C	A	C	A	B
3	C	A	B	B	C	A
4	C	A	B	B	C	A

Any treatment is now preceded equally often by every treatment including itself. This makes the estimation of the contrasts of direct and residual effects orthogonal.

8. Pre-period Change-Over Designs

Pre-period or *0th period* is a period preceding the first experimental period in which appropriate treatments are applied to the experimental unit but either the observations are not recorded or if recorded they are not taken into consideration while carrying out the analysis of the data. By adding a pre-period to the design, the first period observations also receive the residual effects of the treatments and hence the data become homogeneous.

The following is a pre-period design balanced for 4 treatments A, B, C and D:

	Experimental Units			
Periods	1	2	3	4
0	A	B	C	D
1	A	B	C	D
2	D	A	B	C
3	B	C	D	A
4	C	D	A	B

9. Circular Change-Over Designs

A changeover design allowing the estimation of direct and first residual effects with one pre-period, in which the treatments in the period are exactly the same as those in the last period is called a *circular change-over design*.

A circular COD with 7 treatments 14 units and 3 periods with one pre-period for the estimation of direct and first order residual effects is as follows:

		Experimental Units													
		I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV
Periods	0	4	5	6	7	1	2	3	5	6	7	1	2	3	4
	1	1	2	3	4	5	6	7	3	4	5	6	7	1	2
	2	2	3	4	5	6	7	1	6	7	1	2	3	4	5
	3	4	5	6	7	1	2	3	5	6	7	1	2	3	4

10. Method of Construction of a Class of Balanced CODs

The different steps involved in the construction of complete sequence balanced CODs are as follows:

i. Construct two v×v tables in which columns refer to individuals and rows refer to the order of presentation.
ii. In both the squares, number the order of presentation from 1 to v successively.
iii. Number the treatments i = 0, 1, 2,..., v-1.
iv. Assign these treatments successively to the v cells in the first column of both the squares by proceeding from top to bottom, entering only in odd-numbered cells in the first and even numbered cells in the second square, and then reversing the direction, filling in even-numbered cells in the first and odd-numbered cells in the second square.
v. Obtain the successive columns of the squares by adding integer 1 to each element of the previous column and reducing the elements, if necessary, by mod v.

It is to be noted that in each of the constructed squares every treatment occurs in each row and in each column precisely once. Moreover, when v is even, each treatment is preceded exactly once by other treatment in either of the two squares. Thus, in this case either of the two squares may be used. This situation occurs in neither of the two squares if v is odd. However, when both the squares are considered together, each treatment is preceded by every other exactly twice. Consequently, both the squares must be used in this case.

Latin squares for v = 4 are given below:

First square					Second square				
	Experimental Units					Experimental Units			
Period	I	II	III	IV	Period	I	II	III	IV
1	0	1	2	3	1	3	0	1	2
2	3	0	1	2	2	0	1	2	3
3	1	2	3	0	3	2	3	0	1
4	2	3	0	1	4	1	2	3	0

11. Illustration

A change-over design was used at the Poona sheep breeding farm on 12 cross bred sheep of about one year age to compare the effects of protein and non-protein diets on growth of the sheep. Three feeds were tried over 3 periods each of 3 months' duration. The plan and body weight records (lb) of each sheep at the end of each period are given below:

The three treatments are denoted by A, B and C. The basic design has 6 sequences, which have been repeated once more to give 12 sequences in all.

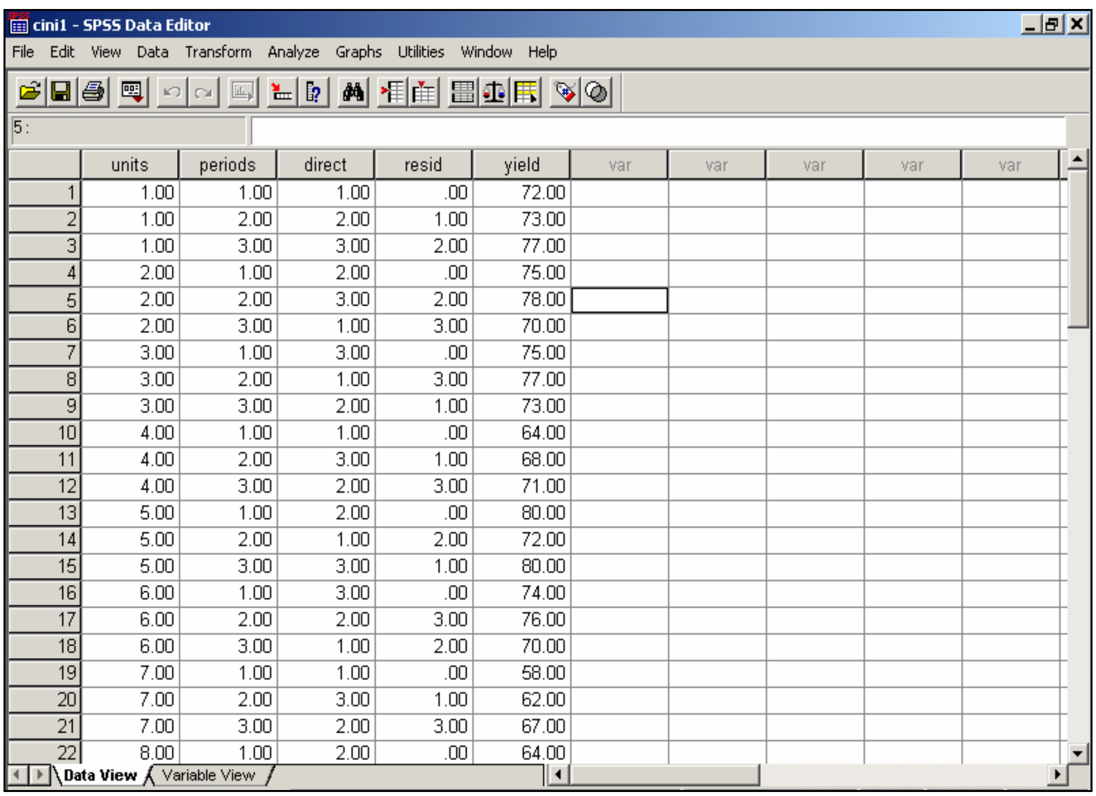
Animal Numbers

Periods	**53**	**54**	**58**	**75**	**81**	**97**
1	A 72	B 75	C 75	A 64	B 80	C 74
2	B 73	C 78	A 77	C 68	A 72	B 76
3	C 77	A 70	B 73	B 71	C 80	A 70
	72	**79**	**106**	**84**	**89**	**70**
1	A 58	B 64	C 72	B 76	A 61	C 71
2	C 62	A 56	B 69	C 79	B 50	A 72
3	B 67	C 60	A 66	A 65	C 60	B 75

Analyse the data using SPSS.

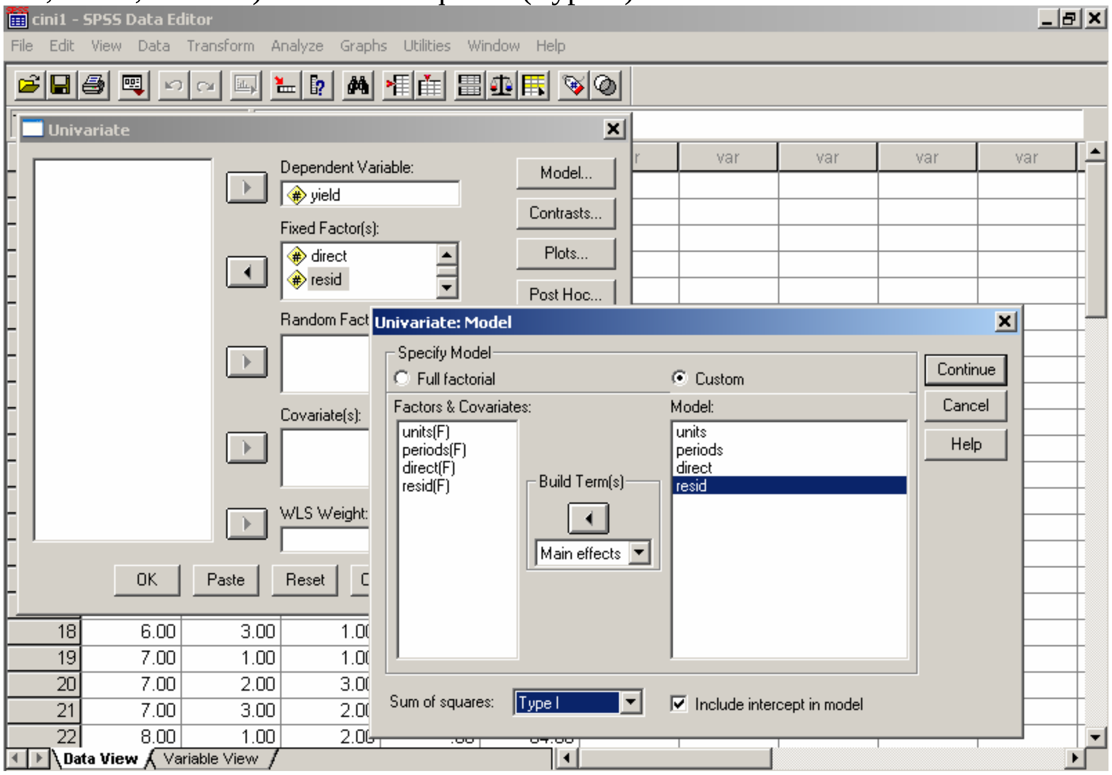
SPSS commands for the analysis of change-over designs
The input data file can be created as shown below:

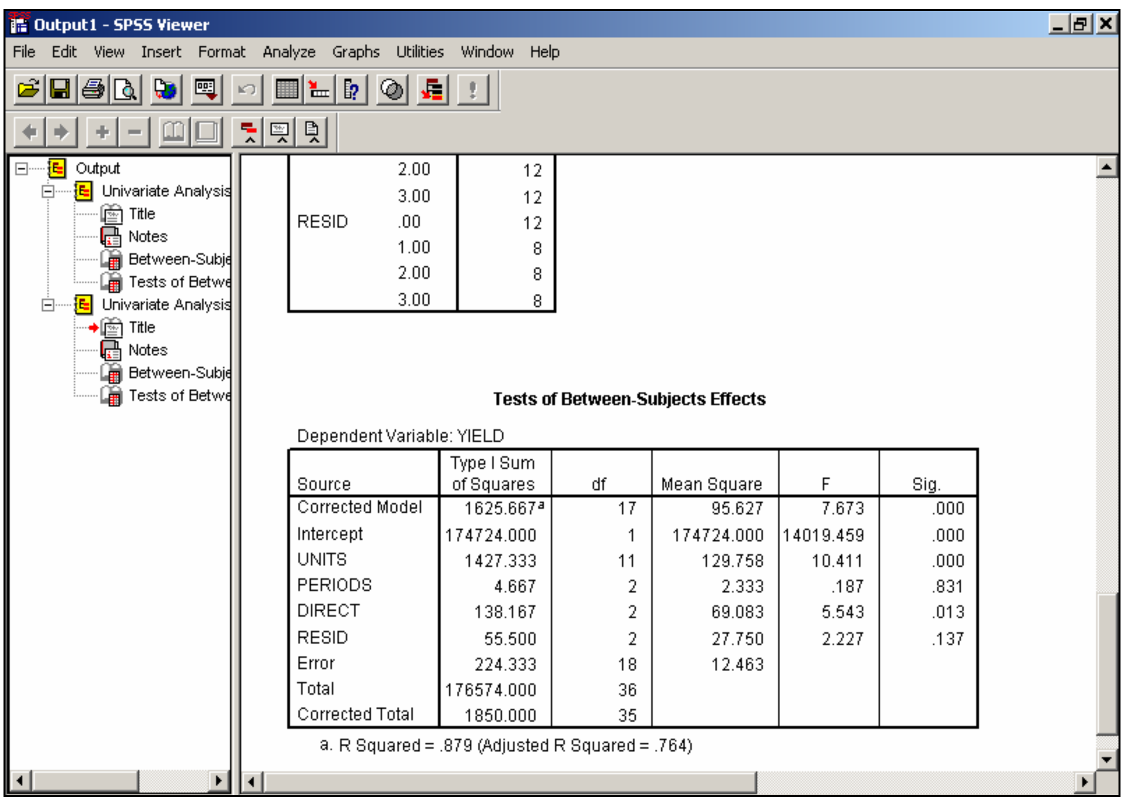
File → New → Data → Variable view → Name (give variable names viz., direct, residual, period, unit, yield) → Data view → Feed data → File → Save (file name).



	units	periods	direct	resid	yield	var	var	var	var	var
1	1.00	1.00	1.00	.00	72.00					
2	1.00	2.00	2.00	1.00	73.00					
3	1.00	3.00	3.00	2.00	77.00					
4	2.00	1.00	2.00	.00	75.00					
5	2.00	2.00	3.00	2.00	78.00					
6	2.00	3.00	1.00	3.00	70.00					
7	3.00	1.00	3.00	.00	75.00					
8	3.00	2.00	1.00	3.00	77.00					
9	3.00	3.00	2.00	1.00	73.00					
10	4.00	1.00	1.00	.00	64.00					
11	4.00	2.00	3.00	1.00	68.00					
12	4.00	3.00	2.00	3.00	71.00					
13	5.00	1.00	2.00	.00	80.00					
14	5.00	2.00	1.00	2.00	72.00					
15	5.00	3.00	3.00	1.00	80.00					
16	6.00	1.00	3.00	.00	74.00					
17	6.00	2.00	2.00	3.00	76.00					
18	6.00	3.00	1.00	2.00	70.00					
19	7.00	1.00	1.00	.00	58.00					
20	7.00	2.00	3.00	1.00	62.00					
21	7.00	3.00	2.00	3.00	67.00					
22	8.00	1.00	2.00	.00	64.00					

Analyze → General linear model → Univariate →Dependent variable (yield) → Fixed factors (units, periods, direct , residual) → Model →Custom → Main effects → Build terms (units, periods, direct, residual) → Sum of Squares (Type I) →Continue → OK.





	2.00	12
	3.00	12
RESID	.00	12
	1.00	8
	2.00	8
	3.00	8

Tests of Between-Subjects Effects

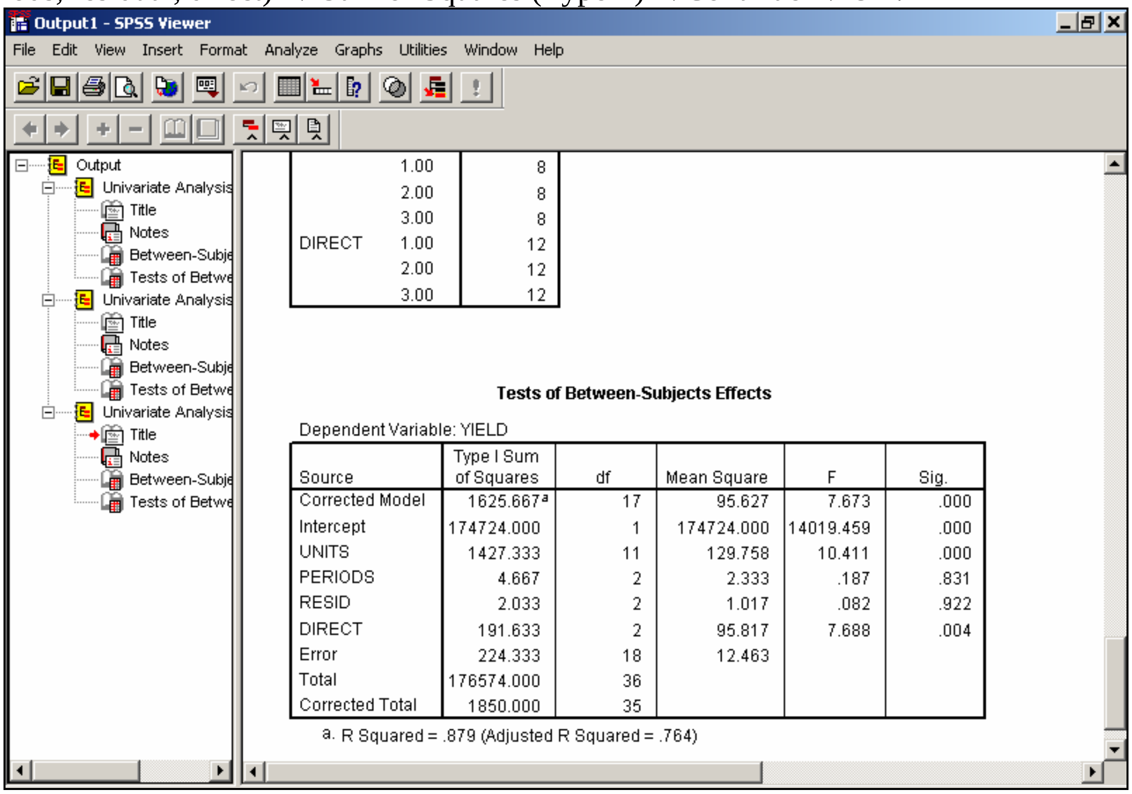
Dependent Variable: YIELD

Source	Type I Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1625.667a	17	95.627	7.673	.000
Intercept	174724.000	1	174724.000	14019.459	.000
UNITS	1427.333	11	129.758	10.411	.000
PERIODS	4.667	2	2.333	.187	.831
DIRECT	138.167	2	69.083	5.543	.013
RESID	55.500	2	27.750	2.227	.137
Error	224.333	18	12.463		
Total	176574.000	36			
Corrected Total	1850.000	35			

a. R Squared = .879 (Adjusted R Squared = .764)

The above analysis gives adjusted SS due to residual effects and unadjusted SS due to direct effects. In order to obtain unadjusted SS due to residual effects and adjusted SS due to direct effects, the following analysis is to be carried out:

Analyze → General linear model → Univariate →Dependent variable (yield) → Fixed factors (units, periods, residual, direct) → Model →Custom → Main effects → Build terms (units, periods, residual, direct) → Sum of Squares (Type I) →Continue → OK.



	1.00	8
	2.00	8
	3.00	8
DIRECT	1.00	12
	2.00	12
	3.00	12

Tests of Between-Subjects Effects

Dependent Variable: YIELD

Source	Type I Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1625.667[a]	17	95.627	7.673	.000
Intercept	174724.000	1	174724.000	14019.459	.000
UNITS	1427.333	11	129.758	10.411	.000
PERIODS	4.667	2	2.333	.187	.831
RESID	2.033	2	1.017	.082	.922
DIRECT	191.633	2	95.817	7.688	.004
Error	224.333	18	12.463		
Total	176574.000	36			
Corrected Total	1850.000	35			

a. R Squared = .879 (Adjusted R Squared = .764)

Final ANOVA Table is obtained by combining both the above as shown below:

ANOVA

Source	D F	S S	M S S	F	Sig.
Units	11	1427.333	129.758	10.411	0.000
Periods	2	4.667	2.333	0.187	0.831
Direct (adj.)	2	191.633	95.817	7.688	0.004
Residual (unadj.)	2	2.033	1.017	0.082	0.922
or					
Direct (unadj.)	2	138.167	69.083	5.543	0.013
Residual (adj.)	2	55.500	27.750	2.227	0.137
Error	18	224.333	12.463		
Total	35	1850.000			

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